Model-based ECG Denoising Using Empirical Mode Decomposition

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Abstract

In this paper, a novel scheme for Electrocardiogram (ECG) denoising is presented based on ECG dynamic model and Empirical mode decomposition (EMD). Firstly, we pre-filter the noisy ECG by making the model fit it in the MMSE sense, in order to preserve the important morphological features, especially the QRS complex. After that, the model is subtracted from the noisy ECG, and the residual signal is then decomposed using EMD and denoised by discarding the noise components from the decomposition results. Finally, the resultant ECG is obtained by combining the model and the denoised residue. Experiments conducted on both real and synthetic ECG data have demonstrated that the proposed method is a superior tool for ECG denoising.

1. Introduction

Electrocardiogram (ECG) signal is a recording of the bioelectric potential produced by rhythmical cardiac activities. ECG has been extensively used for heart disease diagnosis in hospital, as well as patient monitoring at home, since it can provide valuable information of the heart functional conditions. However, reliable and efficient clinical applications are highly dependent on the accuracy of information extracted from the ECG recording, for ECG signals are usually corrupted with various artifacts.

The source of ECG artifacts can be cardiac-related, for example, reduction or disappearance of the isoelectric interval, prolonged repolarization, or atrial flutter; extracardiac noise sources include respiration, changes of electrode position, muscle contraction, and power line interference [1]. Therefore, the goal of ECG denoising is to separate the valid cardiac components from the background noises so as to obtain a signal that allows reliable interpretation.

Due to the overlapping between cardiac components and noncardiac contaminants in frequency, especially from 0.01 Hz to 100 Hz, linear filtering (e.g. low-pass or band-pass filter) is not adequate to eliminate such noises while keeping valid components unchanged [2] [3]. Recently, numerous approaches have been proposed to denoise ECG signals, for example, principal component analysis (PCA) [4], independent component analysis (ICA) [5], neural networks (NN) [6], and wavelet transform (WT) based denoising techniques [7]. Although they demonstrated good performance, the model of ECG in these methods is either fairly arbitrary or essentially based on the frequency content of the ECG and the location of the ECG peaks in time to some degree.

Based on three coupled ordinary differential equations, a dynamic model was firstly developed by McSharry *et al*. [8] for synthesizing artificial ECGs. Later, Clifford *et al.* [9] presented a model-based filtering scheme which fitted the model to a noisy ECG in the MMSE sense by performing a constrained nonlinear optimization. This method can capture much clinical information of the heartbeat, but its efficacy is still limited by the optimization process which easily falls into a local optimal solution.

Recently, a new signal analysis method called Empirical mode decomposition (EMD) has been introduced by Huang *et al*. [10] for analyzing data from nonstationary and nonlinear processes. The major advantage of EMD is that the basis functions used to decompose a signal are not predefined but adaptively derived from the signal itself. Therefore, EMD has found vast applications in signal analysis, including biomedical engineering problems. However, directly applying EMD to the ECG denoising will not produce a desired result due to the physiological characteristics of ECG, as detailed later. In [11], Weng *et al.* proposed to avoid this question by preventing the QRS complex from being filtered, but this method only denoised ECG partly.

In this paper, we propose to apply the EMD algorithm to ECG denoising problem based on the ECG dynamic model. The contribution of our method is that it not only makes use of the advantage of EMD in processing nonlinear and nonstationary signal (e.g. ECG), but also overcomes the potential problem brought by direct ECG denoising with EMD, by utilizing a simple and flexible ECG model to pre-filter the signal. Experiments have been conducted on real ECG records from the MIT-BIH Arrhythmia Database [12] with additive white noise. Also, synthetic ECGs are used to verify the denoising performance when input signal SNR is high. Both quantitative and qualitative results show that our method offers a superior performance for ECG denoising.

2. Background

2.1. Empirical mode decomposition

Empirical mode decomposition (EMD) is intuitive, *a posteriori* and adaptive, with basis functions derived fully from the data. Its essence is to identify the intrinsic oscillatory modes by their characteristic time scales in the signal empirically, and accordingly decompose the signal into intrinsic mode functions (IMFs) by means of a *sifting* process. Therefore, EMD is especially applicable for nonlinear and nonstationary signals, including ECG.

As a counterpart to the harmonic function in Fourier analysis, IMF represents the oscillating mode embedded in the original data. By definition, an IMF should satisfy two conditions: (1) the total number of local extrema and that of zero crossings should be equal to each other or different by at most one, and (2) the mean of the upper and lower envelopes respectively defined by local maxima and local minima should be zero. Based on this definition, *sifting* steps can be summarized as follows:

Given a signal $X(t)$, the first step is to find out all local extrema. Then, all local maxima are connected by a cubic spline line as the upper envelope, and the lower envelope then comes out using the local minima in a similar way. In expectation, the two envelopes can cover all the data between them, and the their mean is designated as $m_1(t)$. So far, the first *sifting* process has been done, and the prototype of the first IMF has come forth as

$$p_1(t) = X(t) - m_1(t) \tag{1}$$

Although $p_1(t)$ should be an IMF ideally, in reality, it still contains more than one extrema between zero crossings due to the overshoots and undershoots involved in the envelope-generating step. Thus, *sifting* has to be repeated on the prototype of IMF until $p_k(t)$ satisfies the two conditions above, and the first IMF $c_1(t)$ is then obtained as the last $p_k(t)$. To terminate the *sifting* process, a common method is accomplished by limiting the size of the standard difference (SD) calculated from the two consecutive sifting results

$$\text{SD} = \sum_{t=0}^{T} \frac{\left|p_{k-1}(t) - p_k(t)\right|^2}{p_{k-1}^2(t)} \tag{2}$$

A typical value for SD can be set between 0.2 and 0.3 [10]. Once $c_1(t)$ is obtained, it is then subtracted from the original data to get a residue $r_1(t)$:

$$r_1(t) = X(t) - c_1(t) \tag{3}$$

Obviously, $c_1(t)$ represents the finest scale mode of oscillation, and $r_1(t)$ still contains useful information about longer time scale components. Therefore, the residue is treated as a new signal, and repeated *sifting* processes are conducted to obtain:

$$r_2(t) = r_1(t) - c_2(t), \quad \cdots, \quad r_n(t) = r_{n-1}(t) - c_n(t) \tag{4}$$

The whole process can be stopped when (1) $c_n(t)$ or $r_n(t)$ is less than a predetermined threshold, or (2) $r_n(t)$ becomes a constant or monotonic function. Combining (3) and (4), we finally get the decomposition result:

$$X(t) = \sum_{k=1}^{n} c_k(t) + r_n(t) \tag{5}$$

where the original signal is decomposed into n IMFs and one residue, which is often denoted as the last IMF $r_{n+1}(t)$ for convenience.

2.2. ECG dynamic model

A realistic synthetic ECG generator was first proposed by McSharry *et al*. [8], using a set of 3-D state equations to produce a trajectory in the Cartesian coordinates. The dynamic equations were then transformed into the polar form for a simpler compact set by Sameni *et al*. [13].

In essence, this model describes each feature wave (e.g. P, Q, R, S and T wave) of one ECG cycle by a Gaussian with three parameters: the amplitude a_i, width b_i, and location θ_i. The vertical displacement of ECG is determined by an ordinary differential equation

$$\dot{z}(\theta) = -\sum_{i \in \{P,Q,R,S,T\}} a_i \frac{\Delta\theta_i}{b_i^2} \exp\left(-\frac{\Delta\theta_i^2}{2b_i^2}\right) \tag{6}$$

where $\Delta\theta_i = (\theta - \theta_i)$ and $\theta \in [0, 2\pi]$, since each cycle of ECG has been linearly mapped into the interval $[0, 2\pi]$ to handle the heart rate variation.

A mathematical representation of one ECG cycle can be obtained by integrating the differential equation (6) with respect to θ

$$z(\theta) = \sum_{i \in \{P,Q,R,S,T\}} a_i \exp\left(-\frac{\Delta\theta_i^2}{2b_i^2}\right). \tag{7}$$

No z-offset exists in this model, for the ECG isoelectric level is assumed as zero. It's obvious that the model approximates an ECG cycle by the sum of five Gaussians with parameters describing wave amplitudes, widths and locations respectively, as shown in Figure 1.

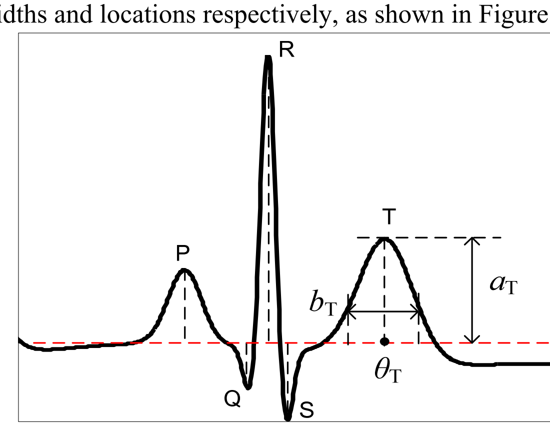

Figure 1. ECG model. Note that the red dashed line denotes the isoelectric level, and only the parameters for T wave are illustrated due to the limited space.

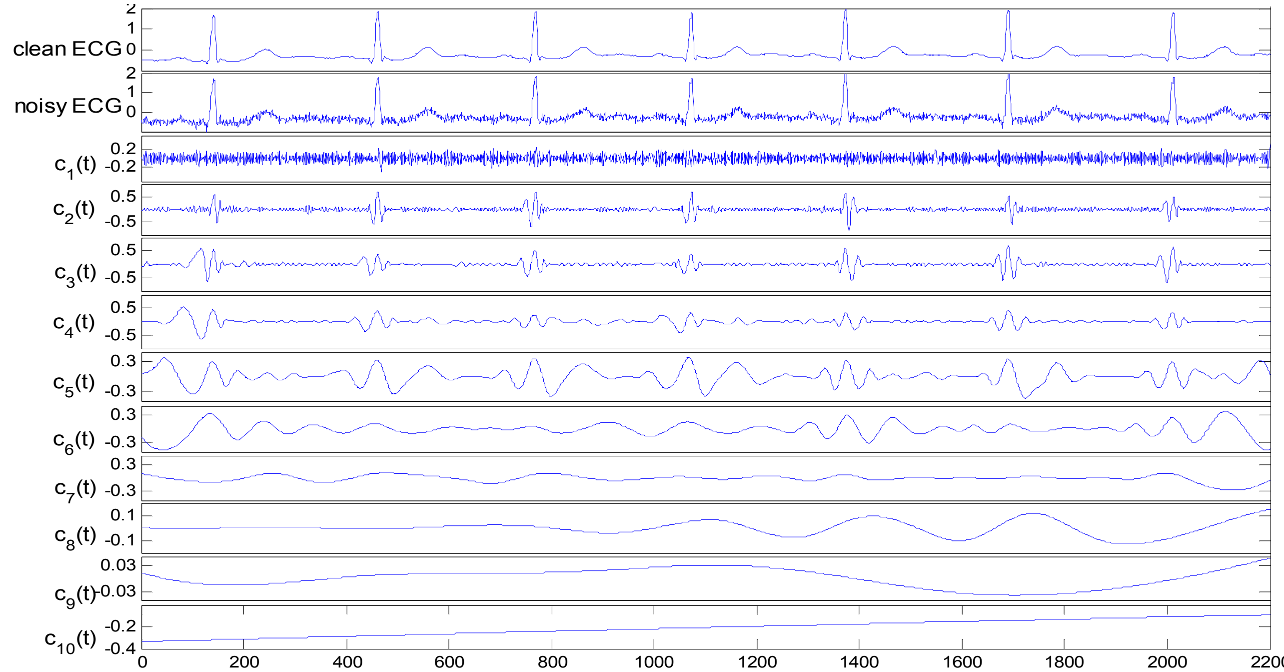

Figure 2. Noisy ECG and decomposition result.

3. Methodology

3.1. Problem of direct ECG denoising by EMD

Since our interest is ECG denoising, a noisy ECG signal is now generated by adding white noise to a clean one, and then decomposed by EMD as shown in Figure 2. The top two subfigures are respectively the clean and noisy ECGs, and below them are all the 10 IMFs. From this figure, we acquire two pieces of information: (1) the average time scale of an IMF increases as the IMF order increases; and (2) the time scale contained in one IMF is not uniform, but various with time.

The basic principle of denoising by EMD is to represent the denoised signal with a partial sum of the IMFs. Although various approaches have been proposed to identify whether a specific IMF contains useful information or noise [14], their performances are not satisfactory when directly applied to the problem of ECG denoising, as shown next.

Examining the IMFs in Figure 2, it's easy to find that the first IMF contains almost nothing but high frequency noise, and that the rest IMFs can be considered to mainly contain useful information about the ECG components, except the second IMF which contains both high frequency noise and components of the QRS complex.

Here comes the dilemma. If we simply discard the first IMF as noise, the output will still consist of considerable noise as illustrated in Figure 3 (a). If we remove the second IMF together, the resultant ECG will have the R waves heavily distorted as shown in Figure 3 (b). Therefore, neither result is satisfactory.

The cause of this problem is that the R wave has a sharp and high waveform, which easily falls in lower order IMFs together with noise components. To deal with this problem, [11] proposed to preserve the QRS complex with a window when discarding the noise-dominant IMFs. Although this method removes much noise from the noisy signal, it suffers a flaw that it has simply ignored the QRS complex in the denoising process, which is of high importance in ECG morphology. Therefore, a more considerate scheme is presented in this paper, by virtue of the ECG dynamic model.

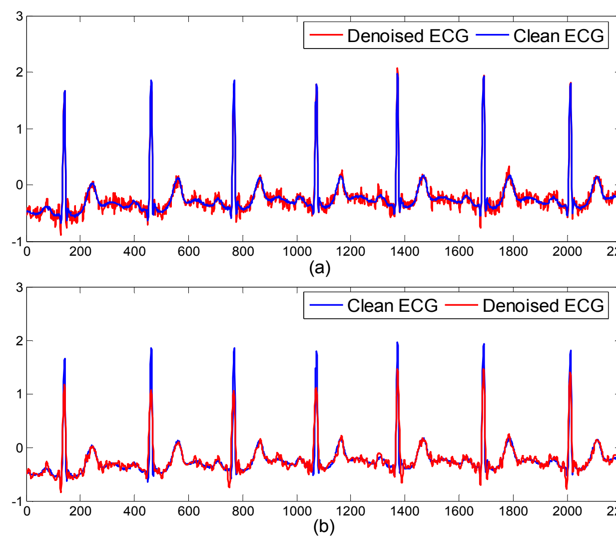

Figure 3. Direct ECG denoising by EMD. (a) Removing the 1st IMF, (b) Removing the top two IMFs. Note the considerable residual noise in (a), and the R wave distortion in (b).

3.2. Model-based pre-filtering

Since the above-mentioned problem originates from ECG characteristics, it should also be solved by them. Fortunately, the ECG dynamic model is an excellent tool to capture ECG morphological features. Clifford *et al.* [9] have demonstrated that this model can be used to filter a noisy ECG by fitting equation (7) to an ECG beat, and the filtered signal can preserve much clinical information, as illustrated in Figure 4 (a). The advantage of this model-based filtering lies in the sound physiological meaning behind the dynamic model.

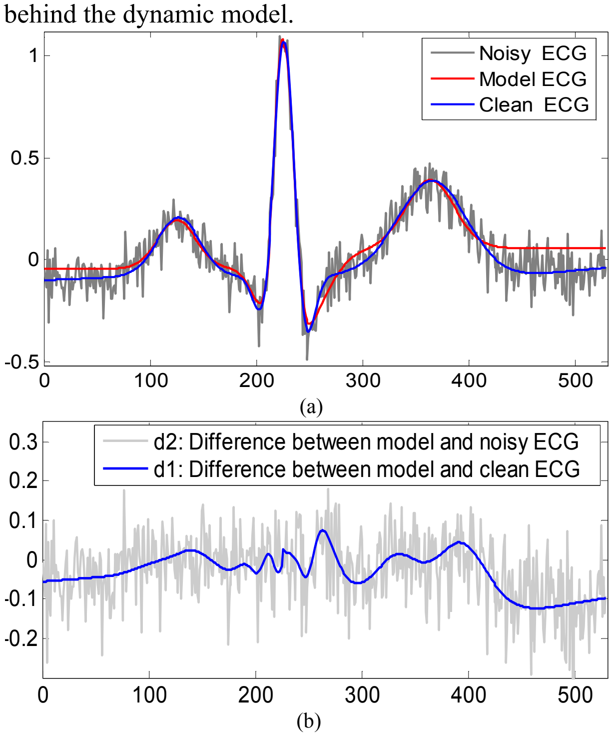

Figure 4. (a) ECG model fitted to noisy ECG, (b) Difference between ECG model and clean ECG.

In practice, the ECG model $z(\theta)$ is fitted to a noisy signal $s(\theta)$ by minimizing the squared error between them. That is, we need to find

$$\min_{a_i, b_i, \theta_i} \left\| z(\theta) - s(\theta) \right\|^2 \qquad (8)$$

over all five $i \in \{P, Q, R, S, T\}$, with $\theta \in [0, 2\pi]$, and solve (8) by a gradient descent method in the parameter space. The Matlab function *lsqnonlin.m* can be applied to implementing this nonlinear optimization. However, this optimization problem can only find a local solution neighboring to the initial solution, so the resultant model usually does not fit the noisy signal well enough, and consequently differs from the clean ECG. As shown in Figure 4 (b), the difference d_1 between model and clean ECG is of low frequency, while the difference d_2 between model and noisy ECG still contains significant noise. From d_2, we wish to estimate the ideal difference d_1, and combine it with the model to obtain the finally denoised ECG, hopefully close to the clean one as much as possible.

3.3. Denoising by EMD

Since the fitted model has filtered and preserved the QRS complex very well as shown in Figure 4 (a), we only need to deal with the residue signal d_2, which is now ready to be denoised by EMD.

Firstly, d_2 is decomposed into a set of IMFs, as shown in Figure 5. In order to obtain the denoised signal as a partial sum of the IMFs, we need to identify and discard IMFs that mainly contain noise components, and then add up all the other IMFs.

In order to determine which IMF comprises useful information and which is primarily of noise, we adopt a significance IMF test procedure, which is proposed by Flandrin *et al.* [15] through analyzing the EMD of fractional Gaussian noise. They found that apart from the first IMF of the noise-only signal, the power spectra of the other IMFs exhibit self similar characteristics like those appearing in any dyadic filter structure. Therefore, the logarithm of the k-th IMF energy E_k, i.e. $\log_2 E_k$, should linearly decrease with respect to k.

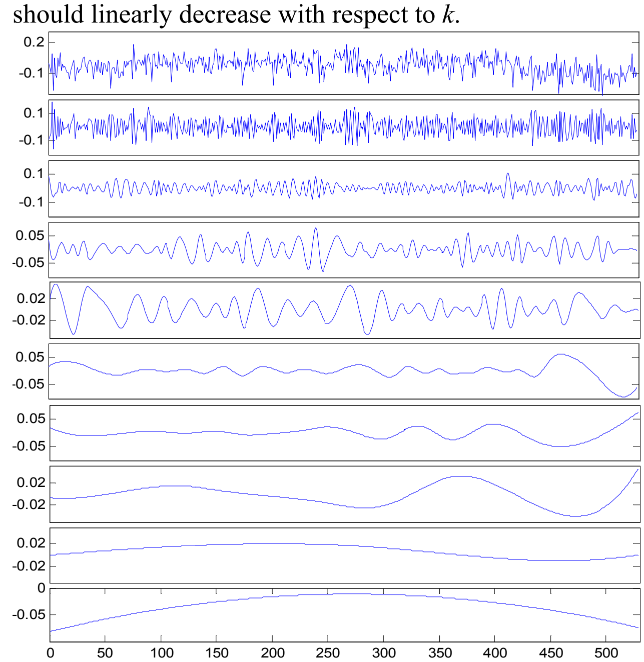

Figure 5. d_2 and its IMFs. The top subfigure is d_2, and others are all its IMFs.

Since we focus on white Gaussian noise (a special case of fractional Gaussian noise) in this paper, the IMF energies of a white-noise-only signal can be described approximately by:

$$E_k = (\sigma^2 2.01^{-k}) / 0.719 \qquad k = 2, 3, 4 \cdots \quad (9)$$

where σ^2 can be approximated by the variance of the first IMF of the noisy signal [16]. In Figure 6, the dashed line shows the relationship between $\log_2 E_k$ and k for a white noise model, while the solid line denotes the relationship for d_2. The first 4 IMFs of d_2 share similar energy distributions with those of the noise-only signal, but from k=5, they diverge significantly from each other, indicating that the top 4 IMFs are primarily noise components and useful information is supposed to reside in the rest IMFs.

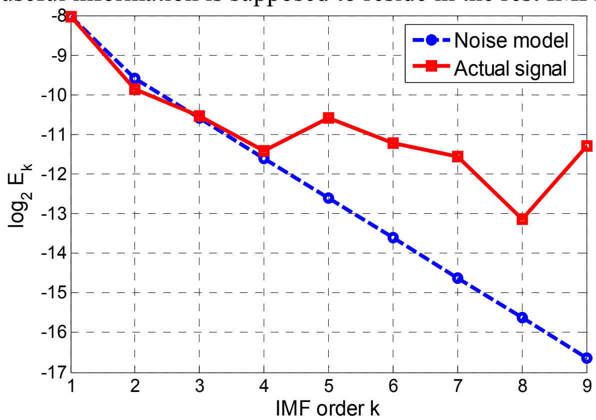

Figure 6. Significance IMF test.

To verify this, the partial sum of the last 5 IMFs is shown in the lower subfigure of Figure 7. Despite of some difference from the ideal signal d_1, the partial sum has captured major information, especially the trend. Finally, we obtain the denoised ECG by adding the partial sum to the model (see section *B*), and the result turns out to be rather closer to the clean ECG than the model, as illustrated in the upper of Figure 7.

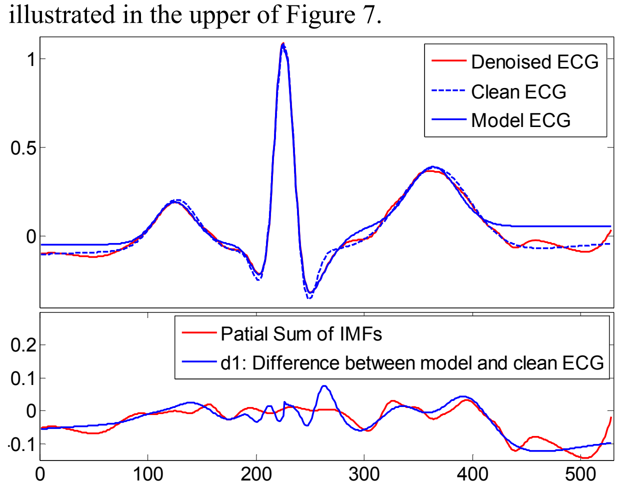

Figure 7. Final denoising results. The upper shows the finally denoised ECG, and the lower shows the partial sum of IMFs of d2.

In order to measure the similarity of IMF energy distribution between the noisy signal and the noise-only one, we can set a difference threshold for $\log_2 E_k$. That is, if the difference exceeds the threshold, we regard the corresponding IMF as a noise component and discard it. In this paper, we empirically choose the threshold as $|0.05\log_2\sigma^2|$, where σ^2 is the variance of the first-order IMF.

4. Experiment results

Experiments are conducted on MIT-BIH Arrhythmia Database [12], and artificial synthetic ECGs. To evaluate the denoising performance, we adopt the notion of Signal-to-noise ratio improvement (*SNRImp*) defined as

$$SNRImp[dB] = SNR_{output} - SNR_{input}$$
$$= 10\log\left(\sum_i |x_n(i) - x_c(i)|^2 \Big/ \sum_i |x_d(i) - x_c(i)|^2\right) \quad (10)$$

where x_c denotes the clean signal, x_n represents the noisy signal, x_d is the denoised one.

To set a benchmark, we have also implemented the window-based EMD denoising scheme which employs a window to preserve the QRS complex. It's worth noting that in this scheme we adopt the significance test procedure (see Section III *C*) to identify noise-dominant IMFs instead of the t-test method used in [11].

4.1. Test on real ECG

We arbitrarily choose ECG records from the database as clean signals, and add white Gaussian noise to them to generate noisy ECGs. Then, we utilize both the proposed method (M-EMD) and the benchmark method (W-EMD) to denoise them. Denoising results *SNRImp* are listed in Table 1, where input SNRs include 15dB, 9dB and 3dB.

As we can see from Table 1, the *SNRImp* increases as the input SNR decreases, for both "W-EMD" and "M-EMD". Moreover, the proposed method produces higher *SNRImp*s than the window-based method does, especially when the input ECG SNR is low (e.g. 3dB).

Table 1. SNRImp(dB) for real ECG

Record No.	W-EMD			M-EMD		
	15dB	9dB	3dB	15dB	9dB	3dB
100	4.13	6.30	7.79	5.05	8.09	9.82
106	4.27	6.44	7.65	5.39	7.88	9.73
123	4.06	7.09	8.15	5.62	8.61	10.89
220	4.52	6.88	7.53	5.82	8.17	9.37
230	5.20	7.13	7.82	6.29	8.33	9.29
Mean	**4.44**	**6.77**	**7.79**	**5.63**	**8.22**	**9.82**

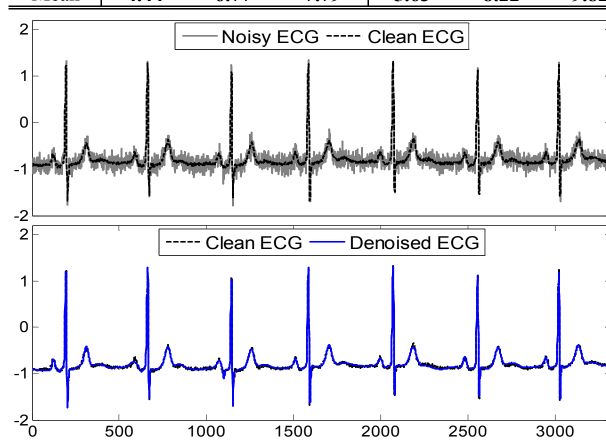

Figure 8. Denoising result for Record 123.

In Fig 8, a portion of Record 123 from the database is illustrated, together with the relevant 9dB noisy ECG and the denoised result using our method. As it's shown, the resultant ECG tracks the clean one closely and smoothly, producing a *SNRImp* 8.61dB, which means that the output SNR has been increased to 17.61dB.

However, a close examination of the so-called clean ECG reveals that it's not as clean as expected, containing considerable high frequency fluctuations. Consequently, interpretation of *SNRImp*s obtained above would suffer questions, especially when input SNR is high. For example, in experiments with 24 dB input, the *SNRImp* is negative sometimes, though the resultant ECG has smoothly tracked the clean one. Since a negative *SNRImp* indicates an inferior result after denoising, it fails to reflect the actual case accurately. To explore the performance in high-input-SNR situations, we carry out another test using synthetic ECG in the next section.

4.2. Test on synthetic ECG

The synthetic ECGs are generated using *ECGSyn* [17], and then contaminated by white Gaussian noise as before. Since we mainly focus on high-input-SNR cases, the SNRs of noisy ECGs are ranged from 15dB to 30dB. As shown in Table 2, the *SNRImp*s for such high-SNR inputs are still positively large, which demonstrates the desirable performance of our method. In the lower of Figure 9, the denoising result is so close to the clean ECG that they can hardly be discerned with unaided eye.

Table 2. SNRImp(dB) for synthetic ECG

Input SNR	30	27	24	21	18	15
SNRImp	6.70	8.34	9.96	10.61	10.7	11.38

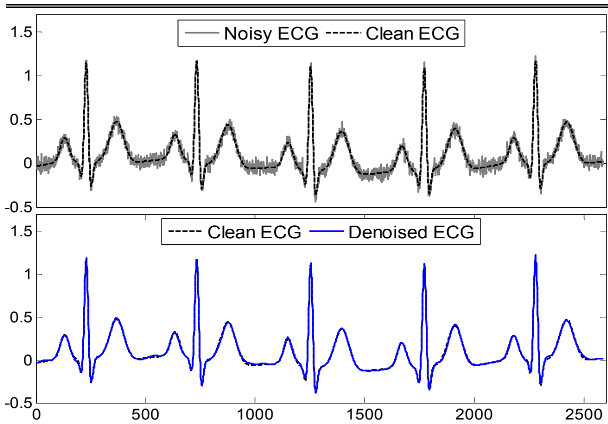

Figure 9. Denoising result for synthetic ECG

5. Conclusion

Empirical mode decomposition (EMD) is an excellent tool for analyzing nonstationary and nonlinear data, and can easily be applied to biomedical signal denoising. However, direct EMD denoising is not suitable for ECG, for the R wave is quite high and sharp, and therefore easily deformed. In this paper, a model-based ECG denoising method using the EMD algorithm is described. Considering the simpleness and flexibility of the ECG model, we use it to pre-filter the ECG and thus preserve the feature waves of ECG, then apply EMD to denoise the residual signal, and finally combine the model and the denoised residue. Experimental results demonstrate that our method can serve as a good tool for ECG denoising.

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